MPGL: An Efficient Matching Pursuit Method for Generalized LASSO Supplemental Material

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Details of the Convex Relaxation

In this section, we provide the details of the convex relaxation for achieving problem (5). Problem (4) in the main paper is a mixed integer programming problem and there are a large amount of feasible $\boldsymbol{\tau}$'s. Thus it is hard to solve. We handle it via a convex relaxation (Tan et al. 2016). To achieve this, we transfer the inner problem of (4) w.r.t. $\mathbf{x}$, $\boldsymbol{\xi}$ and $\mathbf{z}$ into the dual form. Thus we have the following proposition.

Proposition 2. *By introducing dual variables $\boldsymbol{\alpha} \in \mathbb{R}^n$ and $\boldsymbol{\beta} \in \mathbb{R}^l$ to the two constraints, respectively, regarding to the inner problem, problem (4) can be transfered to*

$$\begin{aligned} \min_{\boldsymbol{\tau}\in\boldsymbol{\Lambda}} \max_{\boldsymbol{\alpha}} \ & -\frac{1}{2}\||\boldsymbol{\alpha}\|_2^2 + \boldsymbol{\alpha}^\mathsf{T}\mathbf{y} \\ \text{s.t.} \ & \mathbf{A}^\mathsf{T}\boldsymbol{\alpha} = \mathbf{D}^\mathsf{T}\boldsymbol{\beta}, \ \|\text{diag}(\boldsymbol{\tau})\boldsymbol{\beta}\|_\infty \le \lambda \end{aligned} \tag{13}$$

For any $\boldsymbol{\tau} \in \boldsymbol{\Lambda}$, there is $\boldsymbol{\alpha}^* = \boldsymbol{\xi}^*$ at the optimality of the inner problem.

The proof can be found below.

The values of the components in $\boldsymbol{\beta}$ are bounded by $\|\text{diag}(\boldsymbol{\tau})\boldsymbol{\beta}\|_\infty \le \lambda$. Thus $\boldsymbol{\alpha}$ is bounded because of the equality constraint $\mathbf{A}^\mathsf{T}\boldsymbol{\alpha} = \mathbf{D}^\mathsf{T}\boldsymbol{\beta}$. Without loss of generality, we give a large positive number h and let $\boldsymbol{\alpha} \in [-h, h]^n$. We then define a feasible compact domain of $\boldsymbol{\alpha}$ as $\mathcal{A} = \cap\mathcal{A}_{\boldsymbol{\tau}}$ with $\mathcal{A}_{\boldsymbol{\tau}} = \{\boldsymbol{\alpha}|\mathbf{A}^\mathsf{T}\boldsymbol{\alpha} = \mathbf{D}^\mathsf{T}\boldsymbol{\beta}, \|\text{diag}(\boldsymbol{\tau})\boldsymbol{\beta}\|_\infty \le \lambda, \boldsymbol{\alpha} \in [-h, h]^n\}$ w.r.t. any $\boldsymbol{\tau}$ and

$$\phi(\boldsymbol{\alpha}, \boldsymbol{\tau}) = \frac{1}{2}\|\boldsymbol{\alpha}\|_2^2 - \boldsymbol{\alpha}^\mathsf{T}\mathbf{y}, \ \boldsymbol{\alpha} \in \mathcal{A}_{\boldsymbol{\tau}}. \tag{14}$$

By applying the minimax inequality in (Boyd and Vandenberghe 2004), we have

$$\min_{\boldsymbol{\tau}\in\boldsymbol{\Lambda}} \ \max_{\boldsymbol{\alpha}\in\mathcal{A}} -\phi(\boldsymbol{\alpha}, \boldsymbol{\tau}) \ge \max_{\boldsymbol{\alpha}\in\mathcal{A}} \ \min_{\boldsymbol{\tau}\in\boldsymbol{\Lambda}} -\phi(\boldsymbol{\alpha}, \boldsymbol{\tau}). \tag{15}$$

According to (15), $\max_{\boldsymbol{\alpha}\in\mathcal{A}} \min_{\boldsymbol{\tau}\in\boldsymbol{\Lambda}} -\phi(\boldsymbol{\alpha}, \boldsymbol{\tau})$ is a lower bound of problem (13). By introducing a new variable $\theta \in \mathbb{R}$, we can achieve a convex relaxation of problem (13), a *quadratically constrained linear programming (QCLP)* problem (Pee and Royset 2011):

$$\min_{\boldsymbol{\alpha}\in\mathcal{A},\theta\in\mathbb{R}} \theta, \ \text{s.t.} \ \phi(\boldsymbol{\alpha}, \boldsymbol{\tau}) \le \theta, \forall\boldsymbol{\tau} \in \boldsymbol{\Lambda}.$$

Relying on this relaxation, we arrive problem (4) in QCLP formulation in the main paper.

Proof of Proposition 2

Proof. For any fixed $\boldsymbol{\tau}$, we have the inner problem as

$$\begin{aligned} \min_{\mathbf{x},\boldsymbol{\xi},\mathbf{z}} \ & \frac{1}{2}\|\boldsymbol{\xi}\|_2^2 + \lambda\|\mathbf{z}\|_1 \\ \text{s.t.} \ & \boldsymbol{\xi} = \mathbf{y} - \mathbf{A}\mathbf{x}, \ \mathbf{D}\mathbf{x} = (\mathbf{z} \odot \boldsymbol{\tau}), \end{aligned} \tag{16}$$

By introducing Lagrangian dual variables $\boldsymbol{\alpha} \in \mathbb{R}^n$ and $\boldsymbol{\beta} \in \mathbb{R}^l$ to the equality constraints, the Lagrangian function of the inner problem (16) is:

$$\mathcal{L}(\mathbf{x}, \mathbf{z}, \boldsymbol{\xi}, \boldsymbol{\alpha}, \boldsymbol{\beta}) = \frac{1}{2}\|\boldsymbol{\xi}\|_2^2 + \lambda\|\mathbf{z}\|_1 + \boldsymbol{\alpha}^\mathsf{T}(\mathbf{y} - \mathbf{A}\mathbf{x} - \boldsymbol{\xi}) + \boldsymbol{\beta}^\mathsf{T}(\mathbf{D}\mathbf{x} - \text{diag}(\boldsymbol{\tau})\mathbf{z}) \tag{17}$$

We then minimize $\mathcal{L}(\mathbf{x}, \mathbf{z}, \boldsymbol{\xi}, \boldsymbol{\alpha}, \boldsymbol{\beta})$ w.r.t. $\boldsymbol{\xi}$, $\mathbf{x}$ and $\mathbf{z}$, respectively:

$$\min_{\boldsymbol{\xi}} \ \frac{1}{2}\|\boldsymbol{\xi}\|_2^2 - \boldsymbol{\alpha}^\mathsf{T}\boldsymbol{\xi} + \boldsymbol{\alpha}^\mathsf{T}\mathbf{y} = -\frac{1}{2}\|\boldsymbol{\alpha}\|_2^2 + \boldsymbol{\alpha}^\mathsf{T}\mathbf{y},$$

$$\min_{\mathbf{x}} \ -\boldsymbol{\alpha}^\mathsf{T}\mathbf{A}\mathbf{x} + \boldsymbol{\beta}^\mathsf{T}\mathbf{D}\mathbf{x} = \begin{cases} 0, & \text{if } \mathbf{A}^\mathsf{T}\boldsymbol{\alpha} = \mathbf{D}^\mathsf{T}\boldsymbol{\beta}, \\ -\infty, & \text{otherwise,} \end{cases}$$

$$\min_{\mathbf{z}} \ \lambda\|\mathbf{z}\|_1 - \boldsymbol{\beta}^\mathsf{T}\mathrm{diag}(\boldsymbol{\tau})\mathbf{z} = \begin{cases} 0, & \text{if } \|\mathrm{diag}(\boldsymbol{\tau})\boldsymbol{\beta}\|_\infty \leq \lambda, \\ -\infty, & \text{otherwise,} \end{cases}$$

We obtain $\boldsymbol{\alpha} = \boldsymbol{\xi}$ at the optimum when optimizing $\mathcal{L}$ w.r.t. $\boldsymbol{\xi}$. By substituting above into $\mathcal{L}(\mathbf{x}, \mathbf{z}, \boldsymbol{\xi}, \boldsymbol{\alpha}, \boldsymbol{\beta})$, we obtain the dual of the inner problem

$$\begin{aligned} \max_{\boldsymbol{\alpha}} \ & -\frac{1}{2}\||\boldsymbol{\alpha}\|_2^2 + \boldsymbol{\alpha}^\mathsf{T}\mathbf{y} \\ \text{s.t.} \ & \mathbf{A}^\mathsf{T}\boldsymbol{\alpha} = \mathbf{D}^\mathsf{T}\boldsymbol{\beta}, \ \|\mathrm{diag}(\boldsymbol{\tau})\boldsymbol{\beta}\|_\infty \leq \lambda \end{aligned}$$

This completes the proof.

□

Proof of Proposition 1

Proof. For simplifying the proof, we first reformulate problem (9) by introducing $\boldsymbol{\xi} = \mathbf{y} - \mathbf{A}\mathbf{x}$, subvector $\mathbf{z}_{\mathcal{S}^c}$ w.r.t. $\mathcal{S}^c$ with $\mathbf{z}_{\mathcal{S}^c} = \mathbf{0}$, and one constraint $(\mathbf{D}\mathbf{x})_{\mathcal{S}^c} = \mathbf{0}$. Problem (9) then can be equivalently reformulated as

$$\begin{aligned} \min_{\mathbf{x}, \boldsymbol{\xi}, \mathbf{z}} & \frac{1}{2}\|\boldsymbol{\xi}\|_2^2 + \lambda\|\mathbf{z}_{\mathcal{S}}\|_1, \\ \text{s.t.} \ & \boldsymbol{\xi} = \mathbf{y} - \mathbf{A}\mathbf{x}, \mathbf{z}_{\mathcal{S}^c} = \mathbf{0}, \\ & (\mathbf{D}\mathbf{x})_{\mathcal{S}} = \mathbf{z}_{\mathcal{S}}, (\mathbf{D}\mathbf{x})_{\mathcal{S}^c} = \mathbf{0}. \end{aligned} \tag{18}$$

Following that, before proving Proposition 1, we provide the following lemma.

Lemma 2. *Let $\boldsymbol{\mu} \in \Pi = \{\boldsymbol{\mu} : \sum_{i=1}^t \mu_i = 1, \mu_i \geq 0, \forall i, \boldsymbol{\mu} \in \mathbb{R}^t\}$, problem* (18) *can be solved by the following minimax problem:*

$$\min_{\boldsymbol{\mu} \in \Pi} \max_{\alpha \in \mathcal{A}} \ -\sum_{\boldsymbol{\tau}_i \in \boldsymbol{\Lambda}_t} \mu_i \phi(\boldsymbol{\alpha}, \boldsymbol{\tau}_i). \tag{19}$$

Proof. By introducing the dual variable $\mu_i, \forall i \in [t]$ to the t constraints corresponding to $\boldsymbol{\Lambda}_t$, the Lagrangian function of (6) is $\mathcal{L}(\boldsymbol{\alpha}, \theta, \boldsymbol{\mu}) = \theta - \sum_{i=1}^t \mu_i \phi(\boldsymbol{\alpha}, \boldsymbol{\tau}_i)$. By letting the derivatives w.r.t. θ be zero, we can obtain $\sum_{i=1}^t \mu_i = -1$ at the optimum. Furthermore, we can exchange the order of the $\max$ and $\min$ operators based on the minimax theorem (Sion 1958). This completes the proof. □

Because of the assumption that that there is no overlapping among $\mathcal{C}_i$'s and $\sum_{i=1}^t \mu_i = 1$, problem (19) can be reformulated as:

$$\begin{aligned} \max_{\boldsymbol{\alpha}} \ & -\frac{1}{2}\||\boldsymbol{\alpha}\|_2^2 + \boldsymbol{\alpha}^\mathsf{T}\mathbf{y} \\ \text{s.t.} \ & \mathbf{A}^\mathsf{T}\boldsymbol{\alpha} = \mathbf{D}^\mathsf{T}\boldsymbol{\beta}, \ \|\mathrm{diag}(\boldsymbol{\tau}_i)\boldsymbol{\beta}\|_\infty \leq \lambda, \forall \boldsymbol{\tau}_i \in \boldsymbol{\Lambda}_t \end{aligned} \tag{20}$$

In problem (18), $\mathbf{z}_{\mathcal{S}^c}$ is constrained $\mathbf{z}_{\mathcal{S}^c} = \mathbf{0}$. Based on the assumption that $\mathcal{S}_t = \cup_{i=1}^t \mathcal{C}_i$ and there are no overlapping elements among $\mathcal{C}_i$'s, we have $\|\mathbf{z}_{\mathcal{S}}\|_1 = \sum_{i=1}^t \|\mathbf{z}_{\mathcal{C}_i}\|_1$ and $(\mathbf{D}\mathbf{x})_{\mathcal{C}_i} = \mathbf{z}_{\mathcal{C}_i}, \forall i \in [t]$. By focusing on the subvector $(\mathbf{D}\mathbf{x})_{\mathcal{S}}$ and $\mathbf{z}_{\mathcal{S}}$, the problem (9) can be rewritten as

$$\begin{aligned} \min_{\mathbf{x}, \boldsymbol{\xi}, \mathbf{z}_{\mathcal{S}}} \ & \frac{1}{2}\|\boldsymbol{\xi}\|_2^2 + \lambda \sum\nolimits_{i=1}^t \|\mathbf{z}_{\mathcal{C}_i}\|_1 \\ \text{s.t.} \ & \boldsymbol{\xi} = \mathbf{y} - \mathbf{A}\mathbf{x}, (\mathbf{D}\mathbf{x})_{\mathcal{S}^c} = \mathbf{0}, \\ & (\mathbf{D}\mathbf{x})_{\mathcal{C}_i} = \mathbf{z}_{\mathcal{C}_i}, \forall i \in [t]. \end{aligned} \tag{21}$$

By introducing Lagrangian dual variables $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, the Lagrangian function of the inner problem (21) is:

$$\mathcal{L}(\mathbf{x}, \mathbf{z}_{\mathcal{S}}, \boldsymbol{\xi}, \mathbf{d}, \boldsymbol{\alpha}, \boldsymbol{\beta}) = \frac{1}{2}\|\boldsymbol{\xi}\|_2^2 + \lambda\|\mathbf{z}\|_1 + \boldsymbol{\beta}_{\mathcal{S}^c}^\mathsf{T}(\mathbf{D}\mathbf{x})_{\mathcal{S}^c} + \boldsymbol{\alpha}^\mathsf{T}(\mathbf{y} - \mathbf{A}\mathbf{x} - \boldsymbol{\xi}) + \sum\nolimits_{i=1}^t \boldsymbol{\beta}_{\mathcal{C}_i}^\mathsf{T}((\mathbf{D}\mathbf{x})_{\mathcal{C}_i} - \mathbf{z}_{\mathcal{C}_i}),$$

where $\boldsymbol{\beta}_{\mathcal{C}}, \forall i \in [t]$ and $\boldsymbol{\beta}_{\mathcal{S}^c}$ are subvectors of the $\boldsymbol{\beta}$ to corresponding constraints. To derive the dual form, we minimize $\mathcal{L}(\mathbf{x}, \mathbf{z}_{\mathcal{S}}, \boldsymbol{\xi}, \mathbf{d}, \boldsymbol{\alpha}, \boldsymbol{\beta})$ w.r.t. $\mathbf{x}$, $\boldsymbol{\xi}$ and $\mathbf{z}_{\mathcal{C}_i}, \forall i$:

$$\min_{\boldsymbol{\xi}} \ \frac{1}{2}\|\boldsymbol{\xi}\|_2^2 - \boldsymbol{\alpha}^\mathsf{T}\boldsymbol{\xi} + \boldsymbol{\alpha}^\mathsf{T}\mathbf{y} = -\frac{1}{2}\|\boldsymbol{\alpha}\|_2^2 + \boldsymbol{\alpha}^\mathsf{T}\mathbf{y},$$

$$\min_{\mathbf{x}} \ -\boldsymbol{\alpha}^\mathsf{T}\mathbf{A}\mathbf{x} + \boldsymbol{\beta}_{\mathcal{S}}^\mathsf{T}(\mathbf{D}\mathbf{x})_{\mathcal{S}} + \boldsymbol{\beta}_{\mathcal{S}^c}^\mathsf{T}(\mathbf{D}\mathbf{x})_{\mathcal{S}^c} = \begin{cases} 0, & \text{if } \mathbf{A}^\mathsf{T}\boldsymbol{\alpha} = \mathbf{D}^\mathsf{T}\boldsymbol{\beta}, \\ -\infty, & \text{otherwise}, \end{cases}$$

$$\min_{\mathbf{z}_{\mathcal{C}_i}, \forall i} \ \sum_{i=1}^{t} \left(\lambda\|\mathbf{z}_{\mathcal{C}_i}\|_1 - \boldsymbol{\beta}_{\mathcal{C}_i}^\mathsf{T}\mathbf{z}_{\mathcal{C}_i}\right) \begin{cases} 0, & \text{if } \|\boldsymbol{\beta}_{\mathcal{C}_i}\|_\infty \leq \lambda, \forall i \in [t] \\ -\infty, & \text{otherwise.} \end{cases}$$

By subsisting the above into the Lagrangian function, we prove that problem (20) is the dual form of problem (21), and there is $\boldsymbol{\alpha}^* = \boldsymbol{\xi}^*$ at the optimum. Hence, the subproblem (6) can be addressed by solving (9), and there is $\boldsymbol{\alpha}^* = \boldsymbol{\xi}^*$ and $\boldsymbol{\xi}^* = \mathbf{y} - \mathbf{A}\mathbf{x}^*$ at the optimum. This completes the proof. □

Proofs for the Convergence Analysis

Proof of Lemma 1

Proof. According to definition of problem (5) and subproblem (6), we have

$$\begin{aligned} \theta^* &= \min_{\boldsymbol{\alpha} \in \mathcal{A}} \max_{\boldsymbol{\tau} \in \boldsymbol{\Lambda}} \phi(\boldsymbol{\alpha}, \boldsymbol{\tau}) \\ \text{and} \ \ \theta_t &= \min_{\boldsymbol{\alpha} \in \mathcal{A}} \max_{\boldsymbol{\tau} \in \boldsymbol{\Lambda}_t} \phi(\boldsymbol{\alpha}, \boldsymbol{\tau}). \end{aligned} \tag{22}$$

For any fixed $\boldsymbol{\alpha}$, we have $\max_{\boldsymbol{\tau} \in \boldsymbol{\Lambda}_t} \phi(\boldsymbol{\alpha}, \boldsymbol{\tau}) \leq \max_{\boldsymbol{\tau} \in \boldsymbol{\Lambda}} \phi(\boldsymbol{\alpha}, \boldsymbol{\tau})$, then

$$\min_{\boldsymbol{\alpha} \in \mathcal{A}} \max_{\boldsymbol{\tau} \in \boldsymbol{\Lambda}_t} \phi(\boldsymbol{\alpha}, \boldsymbol{\tau}) \leq \min_{\boldsymbol{\alpha} \in \mathcal{A}} \max_{\boldsymbol{\tau} \in \boldsymbol{\Lambda}} \phi(\boldsymbol{\alpha}, \boldsymbol{\tau}), \tag{23}$$

which means $\theta_t \leq \theta^*$. Furthermore, as t increases, the size of the subset $\boldsymbol{\Lambda}_t$ is increasing monotonically, so $\{\theta_t\}$ is monotonically increasing. The proof is completed. □

Proof of Theorem 1

For convenience, we first define

$$\varphi_t = \min_{i \in [t]} (\max_{\boldsymbol{\tau} \in \boldsymbol{\Lambda}} \phi(\boldsymbol{\alpha}_i, \boldsymbol{\tau})). \tag{24}$$

Furthermore, to complete the proof, we first introduce the following lemma.

Lemma 3. *Let $(\boldsymbol{\alpha}^*, \theta^*)$ be the global optimal solution of (5) in the main paper, and $\{\varphi_t\}$ is corresponding to a sequence $\{\boldsymbol{\alpha}_t, \theta_t\}$ generated by Algorithm 1. There is $\varphi_t \leq \theta^*$, and the sequence $\{\varphi_t\}$ is monotonically decreasing.*

Proof. For $\forall i$, $(\boldsymbol{\alpha}_i, \max_{\boldsymbol{\tau} \in \boldsymbol{\Lambda}} \phi(\boldsymbol{\alpha}_i, \boldsymbol{\tau}))$ is the feasible solution of (5). Then $\theta^* \leq \max_{\boldsymbol{\tau} \in \boldsymbol{\Lambda}} \phi(\boldsymbol{\alpha}_i, \boldsymbol{\tau})$ for $\forall i \in [t]$. Thus we have

$$\theta^* \leq \varphi_t = \min_{i \in [t]} (\max_{\boldsymbol{\tau} \in \boldsymbol{\Lambda}} \phi(\boldsymbol{\alpha}_i, \boldsymbol{\tau})), \tag{25}$$

which shows that, with the increasing iteration t, $\{\varphi_t\}$ is monotonically decreasing. □

Then we conduct the proof for Theorem 1 in the main paper.

Proof. We measure the convergence of the MPGL via the gap between the sequence $\{\theta_t\}$ and $\{\varphi_t\}$. Since $T < \infty$, Algorithm 1 can stop after finite number of iterations. Assume in the t-th iteration in Algorithm 1, there is no update of $\boldsymbol{\Lambda}_t$, i.e. $\boldsymbol{\tau}_t \in \boldsymbol{\Lambda}_t$ and $\boldsymbol{\Lambda}_{t+1} = \boldsymbol{\Lambda}_t$. There will be no update of $\boldsymbol{\alpha}$, i.e. $\boldsymbol{\alpha}_{t+1} = \boldsymbol{\alpha}_t$. Following that, we have

$$\begin{aligned} \min_{\boldsymbol{\alpha} \in \mathcal{A}} \max_{\boldsymbol{\tau} \in \boldsymbol{\Lambda}} \phi(\boldsymbol{\alpha}_t, \boldsymbol{\tau}) &= \min_{\boldsymbol{\alpha} \in \mathcal{A}} \max_{\boldsymbol{\tau} \in \boldsymbol{\Lambda}_t} \phi(\boldsymbol{\alpha}_t, \boldsymbol{\tau}) = \theta_t \\ \text{and} \ \ \varphi_t &= \min_{i \in [t]} \max_{\boldsymbol{\tau} \in \boldsymbol{\Lambda}} \phi(\boldsymbol{\alpha}_i, \boldsymbol{\tau}) \leq \theta_t. \end{aligned} \tag{26}$$

According to Lemma 1 in the main paper and Lemma 3 above in this supplementary material, we have $\theta_t \leq \theta^* \leq \varphi_t$, thus $\theta_t = \theta^* = \varphi_t$ and $\{\boldsymbol{\alpha}_t, \theta_t\}$ is the global optimum of problem (5) in the main paper. □

Figure 3: Visual quality comparison of image deconvolution on `Cameraman`.

More Results on Image Deconvolution

In this section, we show more image deconvolution results for visual comparison. The recovered images in Figure 3, 4, 5 and 6 are corresponding to the images measured in Table 2. The images $\mathbf{x}^*$ and $\mathbf{y}$ are the ground truth image and the input blurry image, respectively. As shown in the results, the proposed MPGL recovers sharper details and clearer background. And there is less ringing artifacts and unnatural textures in our results, which induces the high performance on numerical measurements in Table 2. Conversely, the results of other methods suffer from ringing artifacts and/or over smoothness. Specifically, because of the large motion blur and noise in the input image $\mathbf{y}$ and the periodic boundary assumption, the results of BM3D and FTVd are severely affected by the ringing artifacts.

References

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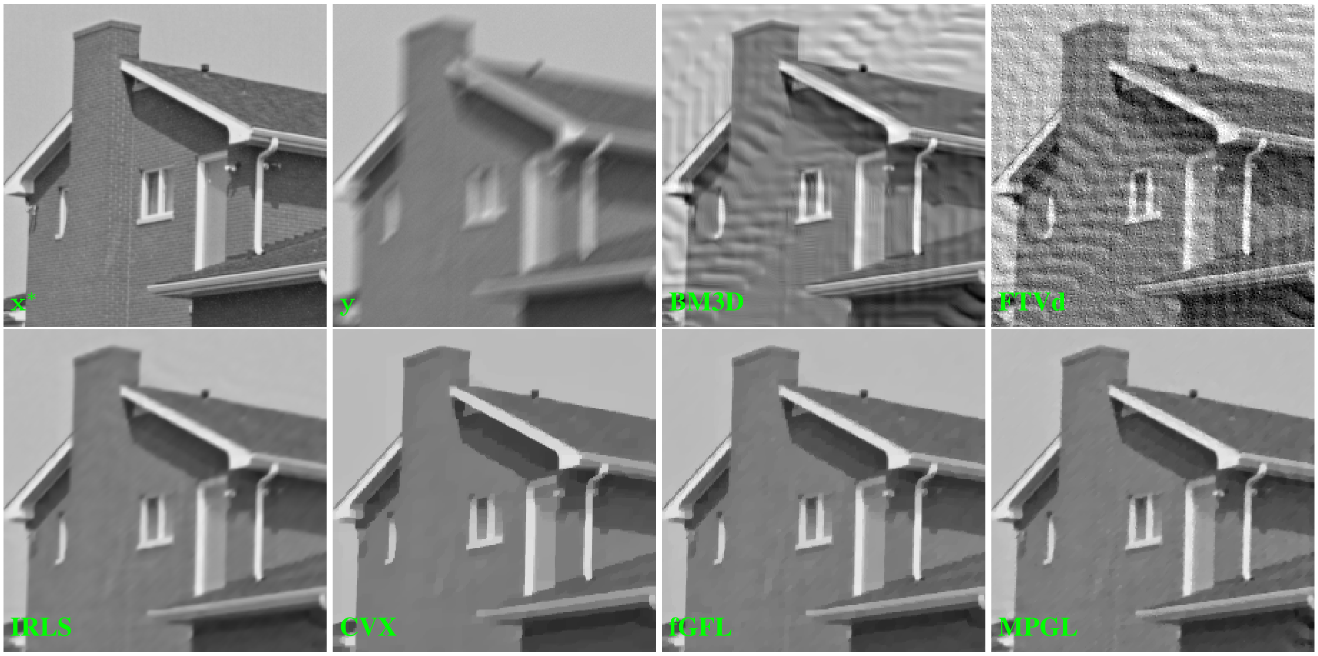

Figure 4: Visual quality comparison of image deconvolution on `House`.

Figure 5: Visual quality comparison of image deconvolution on `Lena`.

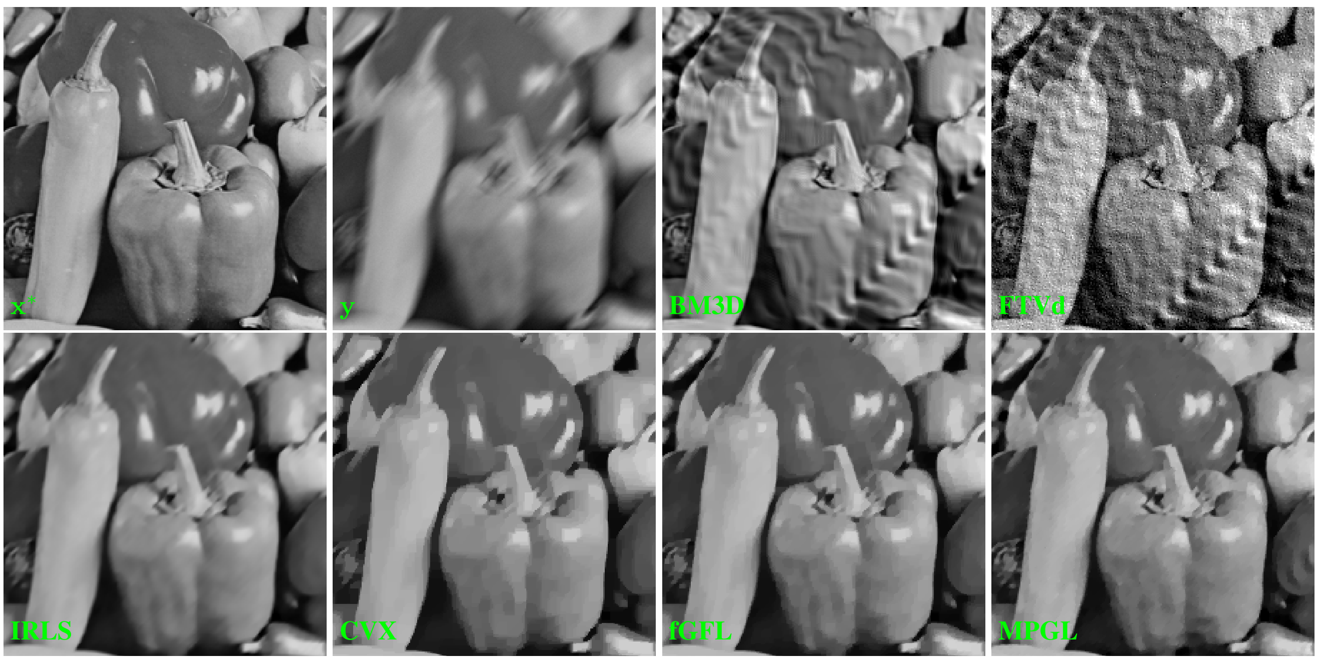

Figure 6: Visual quality comparison of image deconvolution on `Pepper`.